

Schema Redescription in Cellular Automata: Revisiting Emergence in Complex Systems

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Abstract—We present a method to eliminate redundancy in the transition tables of Boolean automata: *schema redescription* with two symbols. One symbol is used to capture redundancy of individual input variables, and another to capture permutability in sets of input variables: fully characterizing the *canalization* present in Boolean functions. Two-symbol schemata explain aspects of the behaviour of automata networks that the characterization of their emergent patterns does not capture. We use our method to compare two well-known cellular automata rules for the *density classification task* [1]: GKL [2], [3] and GP [4]. We show that despite having very different emergent behaviour, these rules are very similar. Indeed, GKL is a special case of GP. Therefore, we demonstrate that it is more feasible to compare cellular automata via schema redescriptions of their rules, than by looking at their emergent behaviour, leading us to question the tendency in complexity research to pay much more attention to emergent patterns than to local (micro-level) interactions.

I. INTRODUCTION

The intersection of biology and computer science has been fertile for some time. Indeed, von Neumann was a member of the mid-twentieth century Cybernetics group [5], whose main focus was the understanding of natural and artificial systems in terms of communication and control. Most early computer science developments were inspired by the models of cognition that orbited this group [6]. Since then, the need to understand how biological systems are able to control and transmit information throughout the huge number of components that comprise them has only increased. The study of complex network dynamics has received much attention in the last two decades. From pioneering work on networks of automata [7]–[9] to recent models of genetic regulation [10]–[13], it is clear that to understand and control the biological organization, it is useful to study models based on complex networks of automata [14]–[16].

There has been much progress in understanding the structure of natural networks—at the level of their topology [17], [18] or of their more fine-grained motifs [19]—as well as on modelling biological systems as networks of automata. Yet, we are still to fully grasp *how* complex networks “compute” and how to harness them to perform specific tasks [20]. The need for a better understanding of collective computation in complex networks has been identified in many areas. For instance, the way plants adjust stomatal apertures for efficient gas exchanges on leaf surfaces is statistically indistinguishable

from the dynamics of automata that compute [21], and the high degree of connectivity in biochemical intracellular signal transduction networks endows them with the capability of emergent nontrivial classification—via collective computation [22]. However, not much work in automata models of complex networks has focused on how interactions at the local level of components are linked to collective behaviour.

We show that eliminating redundancy in the transition tables of local automata reveals the loci of control of collective dynamics, better than via the characterization of collective behaviour, emergent patterns, and the like. Our mechanism to eliminate redundancy is *schema redescription* [23], [24], which we expand here with an additional symbol as explained below. Schemata remove redundancy from automata transition tables, leaving only input states, or groups of input states that *control* state-transitions, known as their *canalizing inputs* [25]. Moreover, we discuss how schema redescription can reveal the links between local and collective behaviour in automata networks (here cellular automata). We exemplify our approach with the classical problem of collective computation in cellular automata: the *density classification task* [1].

II. CELLULAR AUTOMATA

A κ -state automaton is an integer variable $x \in \{0, \dots, \kappa-1\}$, whose value (or state) is computed at time $t+1$ via a state-determined *transition function* F , which takes as input the state of a set of n automata at time t . When $x = 0$ we refer to it as being in the *quiescent* state; any other state is regarded as an *active* state. To study the dynamics of systems of many simple elements (known as complex systems), it is common to study networks of interconnected automata. One way to do this is to arrange automata in a regular lattice of N cells, which we refer to as *cellular automata*, a formalization of a complex dynamical system with its origins in the work of Ulam and von Neumann [26]. In a homogeneous cellular automaton (CA), which we use here, each cell is defined by the same κ -state automaton and is connected to n neighbour cells in the lattice (including itself). Therefore, there are κ^n possible *local neighbourhood configurations* (LNCs). The transition function for every cell, $F = \{f_j\}$, can thus be defined as a *look-up table* (LUT) where each entry, $f_j, j \in \{1, \dots, \kappa^n\}$, is an assignment of a possible input LNC, or *condition part*, to an output state

$x_i(t+1) \in \{0, \dots, \kappa-1\}$. In binary CAs ($\kappa = 2$) it is possible to classify individual cell state transitions in three categories: (1) *preservations*, where a cell does not change its state in the next time step, i.e. $x_i(t) = x_i(t+1)$; (2) *generations*, in which the cell goes from the quiescent to the active state; and (3) *annihilations*, where the cell goes from the active to the quiescent state. The *initial configuration* (IC) of states of a CA lattice is typically random. In the CAs we consider here, cells in the lattice update their states synchronously.

III. SCHEMA REDESCRIPTION

We previously proposed *schema redescription* with one symbol as a method to relate an automaton's LUT to the collective dynamics it produces in one- and two-dimensional CA lattices [23], [24]. Here we introduce two-symbol schemata. The basic idea is simple: LUTs are compressed into parsimonious *schemata* with two extra symbols that capture redundant input states. One symbol captures the *irrelevance* of some inputs in some situations; the other the *permutability* of subsets of inputs. Only the input states that effectively determine the automaton's state transitions remain. A related notion of *canalizing inputs* exists in the context of modelling biochemical networks, e.g. those that regulate the expression patterns of genes. Indeed, automata with canalizing inputs have been shown to stabilize the dynamics of Boolean models of genetic regulation [13], [25], [27].

A. Wildcard Schemata

To unpack the notion of schema, consider a Boolean automaton x , whose state-transition function depends on the states of six Boolean inputs (possibly other automata). Figure 1A depicts the subset of LUT entries (f_α) for automaton x , that lead to state-transition $x(t+1) = 1$.

We first compress LUT entries using schemata that are like LUT entries, but allow an additional *wildcard* ($\#$) symbol (represented graphically in grey) to appear in their condition part. A wildcard input means that *any valid state is accepted for it, with no effect on the state-transition*. This results in a redescription of the set of LUT entries, F , for an automaton x of n inputs into a smaller set of wildcard schemata, $F' \equiv \{f'_v\}$ (see Figure 1B). The set F' , for a given automaton also contains the original entries in its LUT, F , that could not be redescriptioned by wildcard schema. Each specific wildcard schema f'_v redescriptiones a subset of entries in the original LUT, denoted by $\Upsilon_v \equiv \{f_\alpha : f_\alpha \rightarrow f'_v\}$ (\rightarrow means "is redescriptioned by"). Wildcard schemata are *minimal* in the sense that none of the (non-wildcard) inputs in the condition part of a schema can be "raised" to the wildcard status and still ensure the automaton's transition to the same state. Because wildcard schemata are minimal, it follows that for any two wildcard-schemata f'_v and f'_ϕ , $\Upsilon_v \not\subseteq \Upsilon_\phi \wedge \Upsilon_\phi \not\subseteq \Upsilon_v$. Because wildcard schemata are *minimal* and *unique*, they are equivalent to the set of *prime implicants* obtained during the first step of the Quine & McCluskey Boolean minimization algorithm [29]. In other words, a schema is *unique* in the sense that the subset of LUT entries it redescriptiones is not fully redescriptioned

by any other schema. However, in general $\Upsilon_v \cap \Upsilon_\phi \neq \emptyset$. This means that schemata can overlap in terms of the LUT entries they describe. In Figure 1, $\Upsilon_1 \equiv \{f_1, f_5, f_9, f_{13}\}$ and $\Upsilon_9 \equiv \{f_4, f_5, f_6, f_7\}$, therefore $\Upsilon_1 \cap \Upsilon_9 \equiv \{f_5\}$.

Our redescription methodology derives from the work of John Holland on condition/action rules to model inductive reasoning in cognitive systems [28]. However, the same idea had been developed previously for the minimization of circuits in electrical engineering [29]. It was also used by Valiant [30] when introducing *Probably Approximately Correct* (PAC) learning.

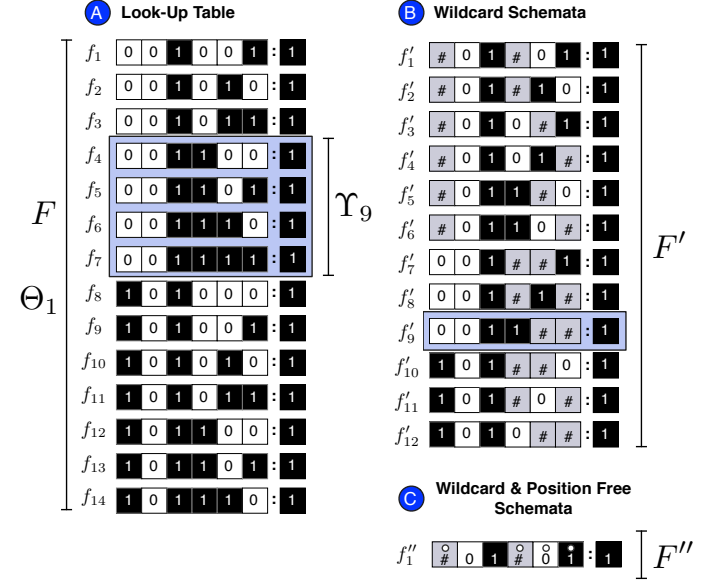


Fig. 1. A set of LUT entries with transition to 1, redescriptioned. White (black) states are 0 (1). Wildcards are grey. Notice that $\Upsilon_9 \equiv \{f_4, f_5, f_6, f_7\}$. Using the additional position-free symbol, the entire set F' is compressed into a single two-symbol schema: f''_1 . Any permutation of the inputs with the position-free symbol results in a schema in F' . Since there is only one set of marked inputs, the position-free symbol does not require an index.

A wildcard schema can always be expressed as a logical conjunction of literals (logical variables or their negation). Since such a schema is a *prime implicant*, it follows that all of its literals are *essential* to determine the automaton's state transition. We refer to the literals (input states) in a wildcard schema as its *essential input states* or *enputs* for short. For instance, schema f'_9 of the example of figure 1 has 4 enputs: the first 4 cells in its condition part. The set F' can be expressed as a logical *disjunctive normal form* (DNF)—that is, a disjunction of conjunctions.

B. Two-Symbol Schemata

Here we introduce a further redescription of F' that results in a set of two-symbol schemata $F'' \equiv \{f''_\theta\}$ (Figure 1C). The *position-free symbol* (\circ_m) above inputs in a condition part means that *any pair of inputs thus marked can "switch places" without affecting the automaton's state-transition*. The index of the position-free symbol, when necessary, is used to differentiate among distinct subsets of inputs whose

elements can only switch places with each other. A two-symbol schema f''_θ redescribes a set $\Theta_\theta \equiv \{f_\alpha : f_\alpha \mapsto f''_\theta\}$ of LUT entries of x . In the same way a wildcard schema captures a subset Υ_v of LUT entries, a two-symbol schema redescribes a subset of wildcard schemata¹. Therefore f''_θ captures the degree of *symmetry* or *group invariance* in the set of wildcard schemata contained in Θ'_θ [31]. We refer to the subsets of such marked inputs in a two-symbol schema as *group-invariant inputs*, which may include wildcard symbols marked with the position-free symbol. Since each two-symbol schema compresses a number of wildcard schemata, it follows that for a given automaton x , $|F''| \leq |F'| \leq |F| = 2^k$.

After redescription, all redundancy is removed in the form of the two symbols. In a given schema, a wildcard input alone is not essential by definition: it is never an *input*, while an individual input set up to a Boolean state always is. However, when a wildcard input is marked with a position-free symbol, then it is part of a group-invariant input defined by the set of inputs thus marked. In this case, the input identifies a group invariance property, such as “*as long as two of these inputs are false*”. To summarize, the set of wildcard schemata F' of an automaton x captures its inputs, while the addition of the position-free symbol captures group invariant relationships in subsets of wildcard schemata. In our working example, the resulting two-symbol schema (Figure 1) reveals that the automaton’s transition to *true* is determined only by a subset of its six inputs: *as long as inputs 2 and 3 are false and true, respectively, and among the others at least one is true and another is false, the automaton will transition to true*. Note that such minimal input constraints are not obvious by looking at the LUT of the example automaton.

In previous work, we used wildcard schemata to identify *conceptual properties* of known solutions to a well-studied problem that requires non-trivial collective computation: the *density classification task* in homogeneous cellular automata. By uncovering these properties, we were able to devise a genetic algorithm that operated on a search space of wildcard schemata, rather than the space of LUTs. We found that the space of schemata is much more amenable to search, with very good correlations among good solutions to the problem. This way, we found some of the best solutions to this problem [23], [24]. Our aims here are: (1) introduce two-symbol schemata; (2) demonstrate that they are more useful to explain and control the function and behaviour of CAs than existing techniques to analyse collective behaviour, and (3) it is more feasible to compare automata transition functions via their schemata redescrptions, than by looking at their emergent or collective behaviour.

¹Given a two-symbol schema, f''_θ , the wildcard schemata it redescribes can be denoted by the set Θ'_θ . It is useful to look at set Θ'_θ as a matrix where rows correspond to wildcard schemata, and columns to their inputs. The last column is the state transition of f''_θ , and may therefore be omitted. The wildcard-schemata rows of matrix Θ'_θ have the same number of zeroes, ones and wildcards. For any pair of inputs (i, j) in f''_θ that are marked with the same position-free symbol, the corresponding i^{th} and j^{th} columns of Θ'_θ can be switched, and the rows of the resulting matrix reordered to obtain Θ'_θ again.

IV. THE DENSITY CLASSIFICATION TASK (DCT)

The *density classification task* (DCT) is the most studied example of collective computation in CAs. The goal is to find a binary CA that can classify the majority state in a random IC. If the majority of cells in the IC is in the quiescent (active) state, after a number of time steps the lattice should converge to a homogeneous state where every cell is in the quiescent (active) state. Since the outcome could be undecidable in lattices with even number of cells (N), lattices with an odd number of cells are used instead. Devising CA rules that perform this task is not trivial, because cells in a CA lattice update their states based on local neighbourhood information, the states of 7 input Boolean variables in this case. Therefore, this task requires information to be transferred across time and (lattice) space to achieve a correct global classification. The performance of CA rule F , on C initial configurations of length N is denoted by $\mathcal{P}_N^C(F)$. The definition of the DCT used in our studies comes from [32].

V. EXPLANATION IN COMPLEX SYSTEMS

The complex systems community has produced methodologies to characterize space-time dynamics in CAs to study emergent behaviour, often using the DCT as a test-case [20]. The most well-known such methodology is the *computational mechanics* framework (CM) [33], [34], used to extract *regular domains*: repeating patterns in the space-time dynamics, formally defined as sets of regular languages. It has been argued that in order to explain emergent behaviour, we should look at the boundaries between domains, known as *particles*, as this is where information is exchanged in the CA lattice [35]. Therefore, collective behaviour in CA dynamics is characterized by finite catalogues of particles and their interaction rules, which define the emergent computation implemented by a given CA [20], and can be seen to possess some features of linguistic representation, though not in fully symbolic forms [36]. Figure 2 depicts the catalogue of particles for the F_{GKL} rule [2], [3], which we describe below.

Other methods exist to detect regularities in CA space-time dynamics, typically statistical and information theoretical filters that uncover patterns that correspond to the particles found via CM [37], [38]. But when it comes to understanding and controlling collective behaviour, a key issue is concerned with finding the best *explanatory* device to use. Do we focus exclusively on the emergent patterns of collective dynamics (the “*emergentist*” paradigm), as CM and related methodologies do? Or can we gain explanatory advantages by paying more attention to the lower level of local transition functions?

In the remaining of this article we pursue the second approach, towards a *hierarchical* methodology that seeks to link local interactions to global behaviour, ultimately allowing us to better understand and control the collective dynamics of complex systems modelled as automata networks.







Regular Domains			Particle Interactions	
$\Lambda^0 = 0^*$	$\Lambda^1 = 1^*$	$\Lambda^2 = (01)^*$	decay	$\mu \rightarrow \delta + \beta$
Particles (Velocities)			react	$\alpha + \delta \rightarrow \gamma$
$\alpha \sim \Lambda^0 \Lambda^1(0)$ 	$\beta \sim \Lambda^2 \Lambda^0(1)$ 			$\beta + \alpha \rightarrow \eta$
$\gamma \sim \Lambda^0 \Lambda^2(3)$ 	$\delta \sim \Lambda^1 \Lambda^2(-1)$ 		particle annihilation	$\gamma + \eta \rightarrow \alpha$
$\eta \sim \Lambda^2 \Lambda^1(-3)$ 	$\mu \sim \Lambda^1 \Lambda^0(0)$ 			$\gamma + \beta \rightarrow \emptyset_1$
				$\delta + \eta \rightarrow \emptyset_2$

Fig. 2. Catalogue of regular domains, particles (domain boundaries), particle velocities (in parentheses), and particle interactions seen in F_{GKL} 's space time behaviour. The colouring assigned to each particle is then used in Figure 3. The notation $p \sim \Lambda^x \Lambda^y$ means that p is the particle forming the boundary between regular domains Λ^x and Λ^y . (Adapted from [1].)

VI. COMPARING COLLECTIVE DYNAMICS

Most of the known one-dimensional CA rules for the DC were analysed using wildcard schemata in [39]. We found that most of these CAs are *process symmetric*: they classify equally well ICs with majority 0s and with majority 1s. We also found that two of the most well-known high-performance CA rules for the DCT, while observing very distinct collective behaviour, are nonetheless similar at the local LUT level [39]. More specifically, we refer to the human-derived CA F_{GKL} [2] [3], and F_{GP} , derived via genetic programming [4]. Figure 3 and 4 depict the space-time dynamics of F_{GKL} and F_{G} respectively. The performance of these rules on the DCT is $\mathcal{P}_{149}^{10^5}(F_{\text{GKL}}) \approx 0.815$, and $F_{\text{GP}} \mathcal{P}_{149}^{10^5}(F_{\text{GP}}) \approx 0.822$. In the remainder of this article, instead of F_{GKL} we use the full equivalent F_{GKL}' , which is the mirror rule of F_{GKL} . That is, exactly the same CA rule, but where the particles traverse the lattice in the opposite direction, with the same velocities and interactions (see *computationally equivalent* CAs in [39]).

The collective behaviour of F_{GKL}' and F_{GP} is quite distinct while F_{GKL}' , after a short transient, leads to the appearance of three domains, the dynamics of the F_{GP} leads to the appearance of many more (we have identified at least ten [36], see figure 4). Because F_{GKL}' only has three domains, from the CI perspective, its collective dynamics can be characterized and explained with a catalogue of six particles: the permutation without repetition of two out of three possible domains, where order is relevant². Indeed, the collective behaviour of F_{GKL}' has been well predicted by the CM framework, using only six particles [40], [41]. In contrast, F_{GP} produces many more particles; even with just ten domains (it has more), there would be ninety possible particles and many potential interactions. Interestingly, even though the collective dynamics of the two rules is apparently (via the CM framework) completely different, we found that F_{GP} can be easily derived from F_{GKL}' , via a small sequence of operations on the wildcard schemata representation of the latter.

The wildcard schemata for F_{GKL}' are shown in Figure 5. Note that most schemata are state-preserving: all state-changes in this automaton are described by two wildcard schemata (f'_5 and f'_{10}). Generations (annihilations) occur only when a

² $p = d!/(d-2)!$, where d is the the number of domains.

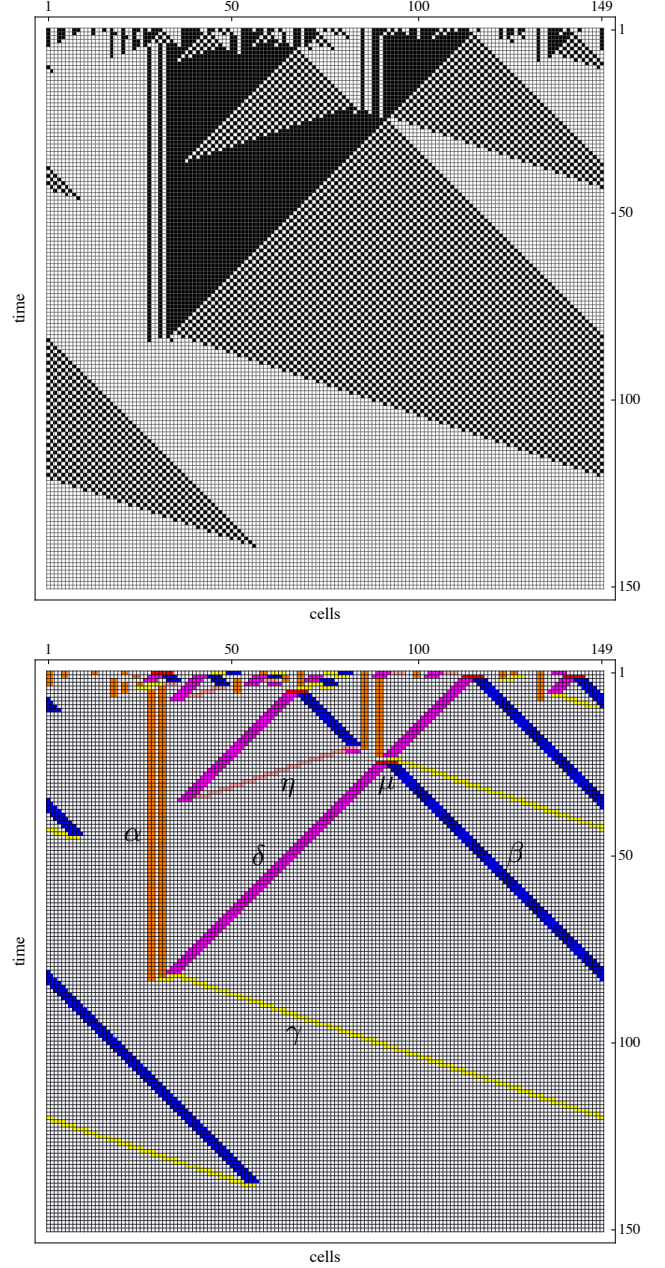


Fig. 3. (Top) A space-time diagram produced by F_{GKL}' . (Bottom) The diagram with the regular domains filtered out, leaving only the locations of the particles (coloured using the scheme in Figure 2). Note that particle α (red) lasts for only one time step, after which it decays to particles δ and β .

cell in state 0 (1) observes right (left) and right-most (left-most) neighbours in the opposite state—this is characterized by schemata f'_5 and f'_{10} . In all other situations the a cell $x_i(t+1) = x_i(t)$.

The wildcard schemata for F_{GP} are shown in Figure 6. In this case, all state changes are characterized by three wildcard schemata: f'_1 , f'_3 , and f'_5 for annihilation, and f'_6 , f'_8 , and f'_{10} for generation. In all other situations the a cell $x_i(t+1) = x_i(t)$. Using the the two-symbol schemata introduced here,

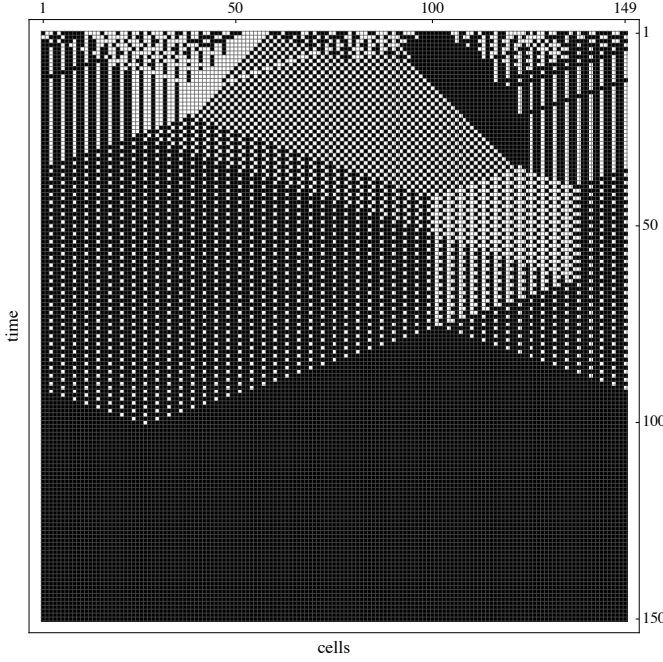


Fig. 4. A space-time diagram produced by F_{GP} .

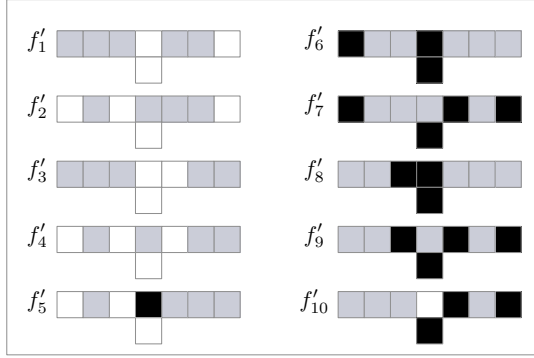


Fig. 5. Wildcard schemata of $F_{GKL'}$. Schemata f'_5 and f'_{10} describe all the possible state-changes for a cell; all other situations are preservations. Notice that schemata f'_2 , f'_4 , f'_7 or f'_9 , when applicable to a state-change, describe LUT entries already described by f'_5 or f'_{10} , respectively; When prescribing preservations the same schemata are captured with less inputs by f'_1 , f'_3 , f'_6 or f'_8 . This means that f'_2 , f'_4 , f'_7 or f'_9 are not essential schemata.

we can appreciate better the similarity between the two rules. Figure 7 depicts the essential state-changing transitions in both rules, which we obtained by via two-symbol schema-redescription, thus removing all redundancy from local dynamics. We can now, for the first time, understand that F_{GP} is doing something very similar to, and which includes the behaviour of, $F_{GKL'}$ at local level.

Both CA rules can be understood by a single schema for generation and their process-symmetric schema for annihilation. Each such schemata is characterized by two inputs. Moreover, one of the inputs is exactly the same for both CAs: for the generation schema, both CAs depend on the right-most neighbour being 1, and for annihilation schemata, both CAs depend on the left-most neighbour being 0.

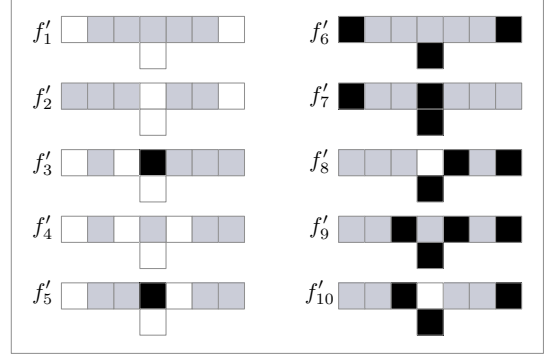


Fig. 6. Wildcard schemata for F_{GP} . f'_1 , f'_3 , f'_5 , f'_6 , f'_8 , and f'_{10} describe all the possible state-changes for a cell x_i ; all other situations are preservations. Notice that schemata f'_4 and f'_9 , when applicable to a state-change, describe LUT entries already described by f'_3 or f'_5 and f'_8 or f'_{10} , respectively

second (group-invariant) input in F_{GP} is also very similar to, and indeed contains the second input for, $F_{GKL'}$. For the generation schema, $F_{GKL'}$ depends on the immediate-right neighbour being 1, and for the annihilation schema, $F_{GKL'}$ depends on the immediate-left neighbour being 0. When it comes to F_{GP} , the second input is a group-invariant input: one of three possible input cells in the neighbourhood may be specified (set to 0 or 1). Specifically, For the generation schema, F_{GP} depends not only on the right-most neighbour being 1, like $F_{GKL'}$, but *alternatively*, on the immediate-left, immediate-right *or* left-most neighbours being 1. Likewise, for the annihilation schema, F_{GP} depends not only on the left-most neighbour being 0, like $F_{GKL'}$, *alternatively*, on the immediate-left, immediate-right *or* right-most neighbours being 0.

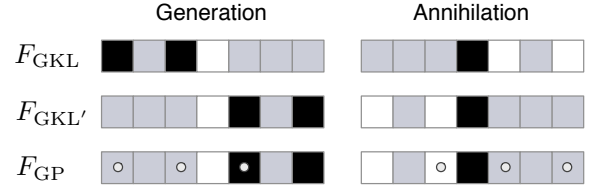


Fig. 7. Two-symbol schemata for $F_{GKL'}$ (F_{GKL}) and F_{GP} , characterizing every possible situation where there are state changes for these rules. Note that here we include the state of the updating cell, (even if not an input in the original schemata) to describe the known state-changing transitions.

VII. CONNECTING THE LEVELS

The similarity between the local-level descriptions of $F_{GKL'}$ and F_{GP} we discovered, as well as their similar performance leads to a natural question: do the additional regular domains produced by the F_{GP} , play a significant role in its collective computation of the DCT? The following three facts are known: (1) F_{GP} produces the same three regular domains as $F_{GKL'}$, plus many others; (2) the performance difference between the two rules, while significant, is less than 1%; and (3) the number of possible particles grows very rapidly with number d of domains ($d!/(d-2)!$). Therefore, it is reasonable to conclude that many—even perhaps most of the regular domains—produced by F_{GP} must be redundant; i.e. the particles they

produce lead to interactions that have no effect on information transmission across the lattice.

Support for this conclusion comes from different sources. One is simple inspection of the space-time dynamics produced by F_{GP} . Figure 8 depicts a portion of the space-time dynamics for F_{GP} shown in Figure 4. Notice, for example, the collision between the particle formed by domains Λ^5 and Λ^3 (highlighted in green), and the particle formed by domains Λ^5 and Λ^6 (orange). This collision, in the CM framework perspective, leads to two new particles: one formed by domains Λ^6 and Λ^4 , and another formed by domains Λ^3 and Λ^4 . However, this collision seems to be redundant, leading to particles that behave exactly as the colliding particles (same velocity, possibly same structure). This is highlighted in Figure 8 by using the same colours for the particles observed before and after the collision. This suggests that the (large) set of possible regular domains in the dynamics of F_{GP} contains sets of domains that are equivalent, such that collisions among the particles they produce are redundant. In the example above Λ^5 and Λ^6 seem to be equivalent, just like Λ^3 and Λ^4 .

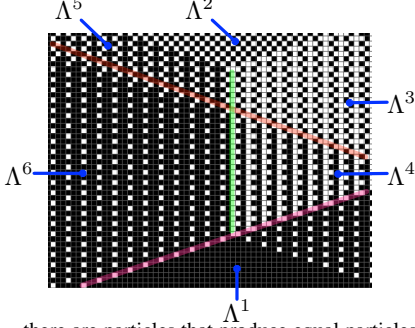


Fig. 8. In F_{GP} there are particles that produce equal particles after collisions (Highlighted in green (vertical) and orange/purple (inclined))

Our observation is supported further when looking at the schemata for F_{GP} . We can use the knowledge encoded in schemata, to better understand the collective behaviour of the CA. Consider f'_1 in F_{GP} , which enforces that a cell x_i in any state whose left- and right-most neighbours are 0 will transition to, or stay 0 at $t+1$ (see Figure 7). This leaves a wide “gap” of five cells in-between the 0-cells, which can be in any state, without affecting the state-transition of x_i to 0. When this pattern is repeated in the lattice, i.e. *every 6th cell is 0*, using the two-symbol schemata obtained instead of the original LUT, we can compute the resulting collective dynamics with *incomplete* information. Indeed, one key advantage of schema redescription is that it allows the simulation of the dynamics of automata networks from ICs where only the state of some cells is specified, while the state of the others is *unknown*.

Figure 9 (A) depicts the *known* dynamics given partial lattice information, where every sixth cell is 0 (except at the lattice borders) and the state of grey cells is *unknown*. This allows us to know how the initial input pattern will propagate in the CA dynamics. In this case, our lack of knowledge propagates in the lattice, and our partial knowledge of the dynamics eventually disappears completely. In 9 (B) we show that different such patterns can co-exist independently, in the

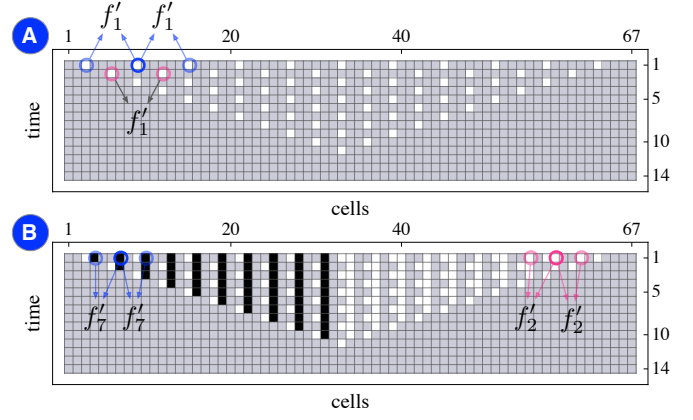


Fig. 9. Dynamically decoupled dynamics of patterns in F_{GP} . See text for description.

previous unknown segments of the lattice. The difference from (A) is that every third cell in the left half of the initial lattice is set to 1, and the third and every third cell after that is set to 0, in the right half of the initial lattice (these conditions match schemata f'_7 and f'_2 respectively).

One very interesting observation from computing dynamics with incomplete information, is that many patterns (such as those in Figure 9) are *dynamically decoupled* in F_{GP} . This means that the space-time patterns produced from schemata f'_1, f'_2 and f'_7 in F_{GP} are updated independently: they function as building blocks that can switch places, producing different domains (from the CM perspective) that in reality are formed of some of the same sub-patterns (defined by the same schemata). This is clear in Figure 9 (B) where the patterns from schemata f'_1, f'_2 and f'_7 , coexist independently of one another. Since some of the schemata for F_{GP} have large “gaps” wherein the states of the cells are irrelevant, if particles collide inside those gaps, some of the decoupled underlying patterns are unaffected, leading to equivalent domains and redundant collisions such as what we apparently observe in Figure 8.

In contrast, rule $F_{GKL'}$ does not easily lead to dynamic decoupled patterns and redundant collisions, since its schemata do not leave large enough “gaps” in the lattice for other patterns to occur. Therefore, $F_{GKL'}$ presents much fewer domains than rule F_{GP} . But since the domains obtained by rule F_{GP} are the result of schemata with a large amount of canalization leading to large “gaps” that can be in any state, many of the obtained distinct domains are dynamically equivalent.

This observation is also supported by the schema redescription of additional, better-performing rules for the DCT: F_{MM0802} [23] and F_{WO} [42], with performances $\mathcal{P}_{149}^{10^5}(F_{MM0802}) \approx 0.845$ and $\mathcal{P}_{149}^{10^5}(F_{WO}) \approx 0.889$. The two-symbol (state-changing) schemata for these rules are depicted in Figure 10. As we can see, for the DCT, increasingly better-performing rules become “less canalized”. This means that they are very sensitive to noise in ICs (as we would expect for the DCT): bit-flips or incomplete state-specification have a high chance of leading to wrong classification (in DCT terms). Since noise is not used in their performance computation and

fitness evaluations, these rules find advantages in becoming increasingly specific to the initial conditions they respond differently to. Therefore, they have a higher number of schemata with more inputs and therefore less canalization to describe their behaviour. The dynamics of these CAs do not produce the large numbers of domains as observed in F_{GP} .³

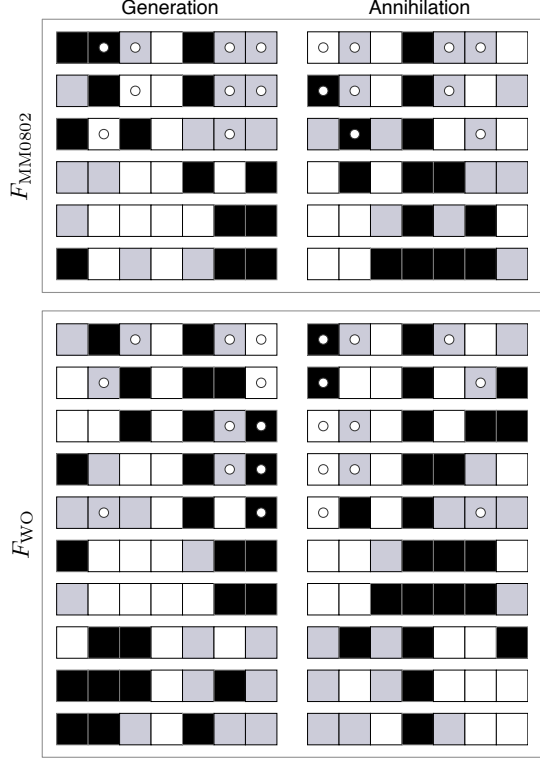


Fig. 10. 2-symbol schemata of F_{MM0802} and F_{WO} , characterizing every possible situation where there are state changes for this rule.

VIII. DISCUSSION

Two-symbol schema redescription allowed us for the first time to understand that despite dramatically distinct collective dynamics (emergent behaviour), the F_{GP} rule is a more general version of the $F_{GKL'}$ rule. The statistically significant, albeit small, performance improvement of F_{GP} over $F_{GKL'}$, is gained by converting a single input, into a group-invariant input. In other words, F_{GP} introduced ambiguity to the dynamical behaviour [43] of $F_{GKL'}$, by allowing the state-transition to optionally occur for a larger set of configurations of a cell's neighbourhood. This redundancy in the form of ambiguity is quite effectively captured by our two-symbol schema redescription with the position-free symbol.

It should be emphasized that until this point, and especially when looking only at the emergent behaviour of these rules, their deep similarity was unknown. With schema redescription of local-level dynamics, not only can we uncover the similarity, but we can explain it. It also makes sense that F_{GP} was

obtained via genetic programming [4]. This rule was probably evolved via minor changes to a tree encoding of F_{GKL} ; it would need simply to substitute a tree terminal for a sub-tree with a disjunction node for 3 terminal edges, plus a mirror operation to go from F_{GKL} to $F_{GKL'}$.

We can also see that both rules are extremely *canalized*: every possible state-change is controlled by two inputs, disregarding the states of all other neighbouring cells. But the F_{GP} is canalizing in yet another way: it depends, alternatively, on the state of one of the input cells in a group-invariant input. These group-invariant (collective) inputs can be formally described as a disjunction of alternative input variables (logical literals) and captured in schemata redescription by the additional position-free symbol. This additional form of canalization in Boolean functions, and its effect on determining collective dynamics in automata networks used to model genetic regulation and signalling networks is a topic we will explore further, elsewhere.

Another key idea we will explore in future work is how to use the explanatory power of schema redescription at the local level of Boolean LUTs, to link local to global behaviour in automata networks. From what we presented here, we can already see that the F_{GP} rule must necessarily lead to some equivalent domains and redundant particle interactions. Since it contains group-invariant inputs, it canalizes distinct and alternative neighbourhood input patterns to the same transition. Therefore, domains that seem to be distinct at the collective space-time behaviour level, can in reality be equivalent and not involved in transmitting novel information through the lattice, merely maintaining the same information in different-looking domains. In a forthcoming paper we will build up from schema redescription at the local level, and the computation of collective dynamics from incomplete information, to automatically identify domains (relevant and redundant), particles, and particle interactions at the collective dynamics level.

As a final topic of discussion, the present work moves us to consider that an exclusively emergentist approach to complex systems dynamics is incomplete. The induction of recurring patterns in dynamics is not a substitute for uncovering the lower-level mechanisms that actually dictate the dynamics. A full account of complex systems requires attention to local and global dynamics. In a sense, too much attention to the “spots” or “stripes” we observe at the emergent level of behaviour, can mask that underneath it all there may be a very similar animal.

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³ redescriptions and dynamics of all the CAs in this paper are available from <http://cnets.indiana.edu/groups/cascl/mrecinca/ca>

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